

northern stations, yet, to attain the maximum shown at 31^d 12^h, the mercury rose 0·40 inch within twelve hours during the 31st of March. I need scarcely point out the weighty bearing which these facts must have on all investigations with reference to the great barometric oscillations within our latitudes as well as to those of lesser magnitude within the tropics.

The following Table contains the daily mean height of the barometer at each station at the hours of maximum and minimum previously given, together with the mean height for the year.

Stations.	1st Max.	Min.	2nd Max.	Mean of year.
	in.	in.	in.	in.
Hobarton	30·087	29·670	29·816	29·794
Pekin	30·131	29·701	29·809 ?	30·015
Cape of Good Hope	30·155	28·848	30·142	30·058
St. Helena	28·309	28·239	28·289 ?	28·296
Makerstoun	29·959	29·770	29·915	29·586
Singapore	29·947	29·864	29·918	29·895
Madras	29·854 ?	29·785	29·844	29·853
Simla	23·184	23·118	23·221	23·195
Catherinenburg	28·967	28·624	29·099	29·023
Bogoslowsk	28·567	28·298	28·708	28·746

It will be perceived that the minimum height was less at all the stations, with the exception of Makerstoun, than the mean for the year.

VI. "On Clairautian Functions and Equations." By Capt. ALLAN CUNNINGHAM, R.E., Hon. Fellow of King's Coll. Lond. (Roorkee, India). Communicated by Prof. CAYLEY. Received April 18, 1876.

(Abstract.)

Notation.—In this paper D stands for $\frac{d}{dx}$; $y', y'' \dots y^{(n)}$ stand for the differential coefficients of y (and therefore y^0 is equivalent to y itself); X, X_1, X_2 , &c. stand for known functions of x ; $X', X'' \dots X^{(m)}$, $X'_1, X''_1 \dots X^{(m)}_1$, &c. stand for the differential coefficients of X, X_1 , &c.; y_m stands for a particular integral of a linear differential equation; y_0 stands for the complete arbitrary portion of the solution of a linear differential equation.

1. *Clairautian Functions.*—It is proposed to apply the term CLAIRAUTIAN FUNCTION to the following expressions (which possess properties similar to that on which the solution of "Clairaut's equation" is founded), viz.

$$y^{(n)}, ky^{(n-1)} - xy^{(n)}, \frac{k(k+1)}{2}y^{(n-2)} + \frac{k}{1} \cdot \frac{(-x)}{1}y^{(n-1)} + \frac{(-x)^2}{2}y^{(n)}, \dots \quad (1)$$

and to denote them by the symbols ${}^kU_{0,n}, {}^kU_{1,n}, {}^kU_{2,n}, \dots {}^kU_{n,n}$, so that

$${}^kU_{r,n} = \frac{k(k+1)(k+2)\dots(k+r-1)}{[r]} y^{(n-r)} + \dots + \frac{k(k+1)(-x)^{r-2}}{[2] [r-2]} y^{(n-2)} + \frac{k(-x)^{r-1}}{[1] [r-1]} y^{(n-1)} + \frac{(-x)^r}{[r]} y^{(n)}, \quad (1)$$

$$= \sum_{p=0}^{p=r} \frac{\Gamma(k+r-p)}{\Gamma(k) \cdot [n-p]} \cdot \frac{(-x)^p}{[p]} y^{(n-r+p)}, \quad (1a)$$

$${}^kU_{n,n} = \frac{k(k+1)(k+2)\dots(k+n-1)}{[n]} y + \dots + \frac{k(k+1)(-x)^{n-2}}{[2] [n-2]} y^{(n-2)} + \frac{k(-x)^{n-1}}{[1] [n-1]} y^{(n-1)} + \frac{(-x)^n}{[n]} y^{(n)} \quad (1)$$

$$= \sum_{p=0}^{p=n} \frac{\Gamma(k+n-p)}{\Gamma(k) \cdot [n-p]} \frac{(-x)^p}{[p]} y^p. \quad (1a)$$

They will be distinguished as of *nth order*, *rth rank*, and *kth class*; *r* and *n* are supposed always *positive integers*, and *r* not $> n$; and *k* may be any quantity whatever. It is obvious that there are

$$\left. \begin{array}{l} (n+1) \text{ Clairautians of } n\text{th order;} \\ (r+1) \text{ terms in a Clairautian of } r\text{th rank in general;} \\ (1-k) \text{ terms in a Clairautian of } r\text{th rank, when } k \text{ is zero or a} \\ \quad \text{negative integer numerically } < r. \end{array} \right\} \quad (2)$$

Thus a Clairautian is a differential expression, the order and rank of which determine the orders of its highest and lowest differential coefficients; it is also obvious that the difference of the exponent (*p*) of *x* and order ($n-r+p$) of differential coefficient is the same in every term.

2. *Clairautian Equations*.—It is proposed to term a differential equation involving Clairautian functions a CLAIRAUTIAN EQUATION. Upon the important properties proved in arts. 4, 5, the solution of many such differential equations may be founded and effected with elegance. These will be developed in what follows.

In consequence of the limitation of *r*, *n* as positive integers (art. 1), the differential equations presented will all be of the ordinary type, that is, involving the differential symbol *D* only in a rational integral form. "General differentiation" will be freely used when necessary to the generality of a solution (so that the quantity *k* may have any value). Any difficulty that may be felt in the interpretation of the transcendent $D^k 0$ (when *k* is not an integer) will generally disappear in the final results, such transcendents, in fact, cancelling.

3. *Algebraic relations*.—It is easy to see that the Clairautians of zero rank ($r=0$) are simple differential coefficients,

$${}^kU_{0,0} = y, \quad {}^kU_{0,1} = y', \quad {}^kU_{0,2} = y'', \quad \dots \dots \dots {}^kU_{0,n} = y^{(n)}; \quad (3)$$

also that those of zero class ($k=0$) are

$${}^0U_{0,n} = y^{(n)}, \quad {}^0U_{1,n} = \frac{-x}{1} y^{(n)}, \quad {}^0U_{2,n} = \frac{(-x)^2}{2} y^{(n)}, \dots \dots \dots$$

$$\dots \dots {}^0U_{n,n} = \frac{(-x)^n}{n!} y^{(n)}. \quad (4)$$

It is easy also to establish by expansion and comparison of coefficients that

$${}^kU_{r,n} = \sum_{p=0}^{p=r} {}^{k-1}U_{p,n-r+p}; \quad {}^kU_{n,n} = \sum_{p=0}^{p=n} {}^{k-1}U_{p,p}, \quad \dots \dots \dots (5)$$

$${}^kU_{r,n} - {}^{k-1}U_{r,n} = {}^kU_{r-1,n-1}; \quad {}^kU_{n,n} - {}^{k-1}U_{n,n} = {}^kU_{n-1,n-1}. \quad \dots \dots (6)$$

4. *Differential properties.*—It is easily established by actual differentiation that

$$D \cdot {}^kU_{r,n} = {}^{k-1}U_{r,n+1}; \quad D^2 \cdot {}^kU_{r,n} = {}^{k-2}U_{r,n+2}; \quad \dots \dots \dots$$

$$D^p \cdot {}^kU_{r,n} = {}^{k-p}U_{r,n+p}. \quad \dots \dots \dots (7a)$$

Hence also, by the theory of “general differentiation,”

$$\left. \begin{aligned} D^{k-p} \cdot {}^kU_{r,n} &= {}^pU_{r,k+n-p}, \quad \dots \dots \dots \\ D^{k-1} \cdot {}^kU_{r,n} &= {}^1U_{r,k+n-1}, \quad D^k \cdot {}^kU_{r,n} = \frac{(-x)^r}{r!} y^{(k+n)}, \quad \dots \dots \dots \end{aligned} \right\} (7b)$$

whatever be the value of k (omitting all arbitrary terms).

These results may be thus expressed in words:—

- 1°. Simple differentiation *depresses the class*, and *raises the order* of a Clairautian, without affecting its rank.
- 2°. All Clairautians of same class (k) and order (n), after k differentiations (in the general sense), contain, omitting arbitrary terms, a common factor $y^{(k+n)}$, and are therefore k th integrals of $y^{(k+n)}$.
- 3°. Hence also all the above quantities may be expressed as integrals of the last of them, ${}^0U_{r,k+n}$, or of $\frac{(-x)^r}{r!} \cdot y^{(k+n)}$.

5. *Symbolic forms.*—It is obvious from results (7), (8), that (omitting arbitrary terms)

$$x^k D^k \cdot {}^kU_{r,n} = \frac{(-1)^r}{r!} \cdot x^{k+r} D^{k+n} y = \frac{(-1)^r}{r!} \cdot x^{k+r} D^{k+r} y^{(n-r)}. \quad \dots \dots (9)$$

Hence, by the known theorem—

$$x^m D^m = \frac{x D}{x D - m}, \quad \text{or} \quad \frac{\Gamma(x D + 1)}{\Gamma(x D - m + 1)} \quad \text{in general,}$$

it follows that

$$\left. \begin{aligned} {}^kU_{r,n} &= \frac{(-1)^r}{[r]} \cdot (x^k D^k)^{-1} \cdot (x^{k+r} D^{k+r}) \cdot y^{(n-r)}, \quad . \quad . \quad . \quad . \quad . \\ &= \frac{(-1)^r}{[r]} \cdot \frac{\Gamma(xD - k + 1)}{\Gamma(xD - k - r + 1)} \cdot y^{(n-r)}, \quad . \quad . \quad . \quad . \quad . \\ &= \frac{(-1)^r}{[r]} \cdot x^k \frac{[xD]}{[xD - r]} \cdot x^{-k} y^{(n-r)}, \quad . \quad . \quad . \quad . \quad . \\ &= \frac{(-1)^r}{[r]} \cdot x^{k+r} D^r \cdot x^{-k} y^{(n-r)}. \quad . \quad . \quad . \quad . \quad . \end{aligned} \right\} (10a)$$

$$\left. \begin{aligned} {}^kU_{n,n} &= \frac{(-1)^n}{[n]} \cdot \frac{\Gamma(xD - k + 1)}{\Gamma(xD - k - r + 1)} \cdot y, \quad . \quad . \quad . \quad . \quad . \\ &= \frac{(-1)^n}{[n]} \cdot x^k \frac{[xD]}{[xD - n]} \cdot x^{-k} y, \quad . \quad . \quad . \quad . \quad . \\ &= \frac{(-1)^n}{[n]} \cdot x^{k+n} D^n \cdot x^{-k} y. \quad . \quad . \quad . \quad . \quad . \end{aligned} \right\} (10b)$$

Observe that, since r, n are supposed positive integers, these symbolic ratios always consist of a finite number of factors (viz. r, n respectively), and are therefore always interpretable, whatever be the value of k .

May 18, 1876.

Dr. J. DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

1. "Absorption-Spectra of Iodine." By Sir JOHN CONROY, Bart., M.A. Communicated by A. G. VERNON HARCOURT, Lee's Reader in Chemistry in the University of Oxford. Received April 12, 1876.

Iodine, as is well known, when in very thin layers, appears red by transmitted light; and when in solution the colour of the liquid depends not only on the amount of iodine contained in it, but also on the nature of the liquid in which it is dissolved.

Schultz-Sellack has pointed out (Pogg. Ann. vol. cxl. p. 334) that the liquids in which iodine is soluble may be divided into two classes:—